ACORN MAGIC:
How Linear Algebra Solves Optimization Problems and Why Do It?

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Abstract—A simple algorithm for solving a set of nonlinear equations by using matrix algebra has been discovered recently—first by transforming them into an equivalent matrix equation and then finding the solutions analytically in terms of the elements of the inverse matrix of this equation. With this newly developed ACORN (an adaptive constrained optimal robust nonlinear) algorithm, it is possible to minimize an objective function without computing its derivatives. The convergence of this nonlinear analytic iterative formula requires the proper values of two control parameters (independent of the problem size). This presentation will describe what ACORN is and how it is used to solve large-scale optimization problems with an innovative approach Acorn Magic (minimization algorithms gathered in a cloud).
Le Equivalent-Matrix Equation

\[ f_i(x_1, x_2, \ldots, x_m) = 1 \text{ for } i = 1, 2, \ldots, m \]

\[ M \vec{x} = \vec{1} \]

\[
\begin{pmatrix}
1 - (m - 1)r & rf_1 & rf_1 & \cdots & rf_1 \\
rf_1 & x_1 & x_2 & \cdots & x_m \\
rf_2 & rf_2 & x_2 & \cdots & x_m \\
\vdots & \vdots & rf_m & \cdots & rf_m \\
rf_m & rf_m & rf_m & \cdots & rf_m
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_m
\end{pmatrix}
= 
\begin{pmatrix}
1 \\
1 \\
1 \\
\vdots \\
1
\end{pmatrix}
\]

with \( r = \text{root control parameter} \) of a fixed-point iteration.
## Lee-Matrix and Its Inverse

The Lee-Matrix $M$ is defined as:

$$
M = \begin{pmatrix}
\frac{1-(m-1)r}{x_1} & \frac{rf_1}{x_2} & \frac{rf_1}{x_3} & \cdots & \frac{rf_1}{x_m} \\
\frac{rf_2}{x_1} & \frac{1-(m-1)r}{x_2} & \frac{rf_2}{x_3} & \cdots & \frac{rf_2}{x_m} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\frac{rf_m}{x_1} & \frac{rf_m}{x_2} & \frac{rf_m}{x_3} & \cdots & \frac{1-(m-1)r}{x_m}
\end{pmatrix}
$$

The inverse of the Lee-Matrix $M(x)$ is given by:

$$
M(x)^{-1} = \frac{1}{1-mr} \begin{pmatrix}
(1-r)g_1x_1 & -rg_2x_1 & -rg_3x_1 & \cdots & -rg_mx_1 \\
-rg_1x_2 & (1-r)g_2x_2 & -rg_3x_2 & \cdots & -rg_mx_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
-rg_1x_m & -rg_2x_m & -rg_3x_m & \cdots & (1-r)g_mx_m
\end{pmatrix}
$$

where

$$
g_k = \frac{1}{f_k}.
$$
Lee Identity-Matrix and Fixed-Point Iteration

\[ M^{-1}M = I \text{ (the identity matrix)} \]

Define Lee Identity Matrix Equation: \( L \bar{x} = M^{-1}M \bar{x} \)

\[
L = \frac{1}{1 - mr} \begin{pmatrix}
g_1 - mrg_{\text{avg}} & 0 & \ldots & 0 \\
0 & g_2 - mrg_{\text{avg}} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & g_m - mrg_{\text{avg}}
\end{pmatrix}
\]

\[
g_{\text{avg}} = \frac{1}{m} \sum_{k=1}^{k=m} g_k
\]

**Fixed-point iteration formula:**

\[
\bar{x}_{k+1} = \frac{g_k - mrg_{\text{avg}}}{1 - mr} \bar{x}_k
\]
Since \( \Delta x_k^{next} = \frac{x_k^{next} - x_k}{x_k} = \frac{(g_k - 1) - mr(g_{avg} - 1)}{1 - mr} \),

\( \Delta x_{avg}^{next} = (g_{avg} - 1) \); When \( g_{avg} \to 1 \), \( \bar{x} \) converges.

Define \( r \) and \( p \) as the root and path convergence control parameter with \( g_k(\bar{x}, p) = \frac{1 + p}{f_k + p} \) of a fixed-point iteration.
How to use ACORN to Solve Optimization Problems?

**Lee Function**

\[ f_k(\bar{x}) = \left( \frac{\partial f_{obj}(\bar{x})}{\partial x_k} \right)^2 + 1 \]

Note--Each partial derivative (fobj) ranging from -\infty to +\infty is transformed to a set of Lee Functions ranging from 1 to 0 by:

1) \( \text{fobj'}^2 \) ranging from 0 to +\infty;
2) \( \text{fobj'}^2 \) to \( \text{fobj'}^2 + 1 \) ranging from 1 to +\infty; and
3) \( \text{fobj'}^2 + 1 \) to its reciprocal ranging from 1 to 0.
Robust goes to zero at end

r goes to zero at end
Le Equivalent Objective Function and The ACORN BOA (Basin-of-Attraction) Plot

$f_k(\bar{x}, r, p) = F_k(f_{\text{obj}}(\bar{x}), \bar{x}, r, p) + 1$, appropriately defined so that when $f_{\text{obj}}(\bar{x})$ converges the Lee Objective Function $F_k(f_{\text{obj}}(\bar{x}), \bar{x}, r, p)$ converges (derivative-free).
NPSLAC: A Brief History of SOL and SLAC 1980-1995

Introduction

The design of a structure to a set of collisions at the major arc of belonging to the capability of operating (hi-η*) modes, where a design which meet has been named the how this design w configurations in
NPSLAC:
A customized version of NPSOL made specially by Dr. Margaret Wright and Dr. Philip Gill for my work in accelerator design, commission, and control at SLAC that led to the purchase of this famous Next Computer!
“Do you have a problem? Not a small problem, but a really big one? Perhaps Dr. Michael Saunders can help you.”


If you like to know the nonlinear nature of your objective function, an ACORN Basin-of-Attraction Plot perhaps can help you.